Exercise 3A

1 $\frac{d}{dx}(\sinh 2x) = 2\cosh 2x$

2 $\frac{d}{dx}(\cosh 5x) = 5\sinh 5x$

3 $\frac{d}{dx}(\tanh 2x) = 2\text{sech}^2 2x$

4 $\frac{d}{dx}(\sinh 3x) = 3\cosh 3x$

5 $\frac{d}{dx}(\coth 4x) = -4\text{cosech}^2 4x$

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$$\begin{aligned}\frac{d}{dx}(\text{sech}\, 2x) &= \frac{-1}{(\cosh 2x)^2} \times 2\sinh 2x \\ &= -2\frac{\sinh 2x}{\cosh 2x} \times \frac{1}{\cosh 2x} \\ &= -2\tanh 2x\ \text{sech} 2x\end{aligned}$$

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$$\begin{aligned}\frac{d}{dx}(e^{-x}\sinh x) &= -e^{-x}\sinh x + e^{-x}\cosh x \\ &= e^{-x}(\cosh x - \sinh x)\end{aligned}$$

8 $\frac{d}{dx}(x\cosh 3x) = \cosh 3x + 3x\sinh 3x$

9

$$\begin{aligned}\frac{d}{dx}\left(\frac{\sinh x}{3x}\right) &= \frac{\cosh x}{3x} - \frac{\sinh x}{3x^2} \\ &= \frac{x\cosh x - \sinh x}{3x^2}\end{aligned}$$

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$$\begin{aligned}\frac{d}{dx}(x^2\cosh 3x) &= 2x\cosh 3x + x^2 \times 3\sinh 3x \\ &= x(2\cosh 3x + 3x\sinh 3x)\end{aligned}$$

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$$\begin{aligned}\frac{d}{dx}(\sinh 2x\cosh 3x) &= 2\cosh 2x\cosh 3x + \sinh 2x \times 3\sinh 3x \\ &= 2\cosh 2x\cosh 3x + 3\sinh 2x\sinh 3x\end{aligned}$$

12 $\frac{\mathrm{d}}{\mathrm{d}x}(\ln\cosh x) = \frac{1}{\cosh x} \times \sinh x$

$= \tanh x$

13 $\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x^3) = 3x^2 \cosh x^3$

14 $\frac{\mathrm{d}}{\mathrm{d}x}(\cosh^2 2x) = (2\cosh 2x)(2\sinh 2x)$

$= 4\cosh 2x \sinh 2x$

15 $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{\cosh x}) = \sinh x \mathrm{e}^{\cosh x}$

16 $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{cosech}x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\sinh x}\right) = \frac{0 - 1 \times \cosh x}{\sinh^2 x}$

$= -\coth x \,\mathrm{cosech}x$

17 $y = a\cosh nx + b\sinh nx$

Differentiate with respect to x

$\frac{\mathrm{d}y}{\mathrm{d}x} = an\sinh nx + nb\cosh nx$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = an^2\cosh nx + bn^2\sinh nx$

$= n^2(a\cosh nx + b\sinh nx)$

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = n^2 y$

18 $y = 12\cosh x - \sinh x$

$$\frac{dy}{dx} = 12\sinh x - \cosh x$$

At the stationary values $\frac{dy}{dx} = 0$

$$12\sinh x - \cosh x = 0$$

$$12\left(\frac{e^x - e^{-x}}{2}\right) - \left(\frac{e^x + e^{-x}}{2}\right) = 0$$

$$\frac{11}{2}e^x - \frac{13}{2}e^{-x} = 0$$

$$\frac{11}{2}e^{2x} - \frac{13}{2} = 0$$

$$e^{2x} = \frac{13}{11}$$

$$2x = \ln\left(\frac{13}{11}\right)$$

$$x = \ln\left(\sqrt{\frac{13}{11}}\right)$$

$$y = 12\cosh x - \sinh x$$

$$= 12\left(\frac{e^x + e^{-x}}{2}\right) - \left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \frac{11}{2}e^x + \frac{13}{2}e^{-x}$$

When $x = \ln\left(\sqrt{\frac{13}{11}}\right)$

$$y = \frac{11}{2}e^{\ln\left(\sqrt{\frac{13}{11}}\right)} + \frac{13}{2}e^{-\ln\left(\sqrt{\frac{13}{11}}\right)}$$

$$= \frac{11}{2}\sqrt{\frac{13}{11}} + \frac{13}{2}\sqrt{\frac{11}{13}}$$

$$= \frac{1}{2}\sqrt{13\times 11} + \frac{1}{2}\sqrt{11\times 13}$$

$$= \sqrt{13\times 11}$$

$$= 11.958...$$

$$= 12.0 \text{ (3 s.f.)}$$

So the turning point is at $\left(\ln\left(\sqrt{\frac{13}{11}}\right), \sqrt{143}\right)$ or (0.0835, 12.0)

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$$y = \cosh 3x \sinh x$$
$$\frac{dy}{dx} = 3\sinh 3x \sinh x + \cosh 3x \cosh x$$
$$\frac{d^2y}{dx^2} = 9\cosh 3x \sinh x + 3\sinh 3x \cosh x + 3\sinh 3x \cosh x + \cosh 3x \sinh x$$
$$= 10\cosh 3x \sinh x + 6\sinh 3x \cosh x$$
$$= 2(5\cosh 3x \sinh x + 3\sinh 3x \cosh x)$$

20 $\frac{x^2}{256} - \frac{y^2}{16} = 1$

$$x^2 - 16y^2 = 256$$
$$2x - 32y\frac{dy}{dx} = 0$$
$$32y\frac{dy}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{x}{16y}$$

At $(16\cosh q, 4\sinh q)$

$$\frac{dy}{dx} = \frac{16\cosh q}{64\sinh q}$$
$$= \frac{\cosh q}{4\sinh q}$$

So the tangent has the equation:

$$y - 4\sinh q = \frac{\cosh q}{4\sinh q}(x - 16\cosh q)$$
$$4y\sinh q - 16\sinh^2 q = x\cosh q - 16\cosh^2 q$$
$$4y\sinh q - x\cosh q = 16\sinh^2 q - 16\cosh^2 q$$
$$4y\sinh q - x\cosh q = -16$$

The normal has gradient $-1 \div \frac{16\cosh q}{\sinh q} = -\frac{4\sinh q}{\cosh q}$

So the normal has equation:

$$y - 4\sinh q = -\frac{4\sinh q}{\cosh q}(x - 16\cosh q)$$
$$y\cosh q - 4\sinh q\cosh q = -4x\sinh q + 64\sinh q\cosh q$$
$$4x\sinh q + y\cosh q = 68\sinh q\cosh q$$